

# Pheromone-based Column Generation for the Vehicle Routing Problem with Black Box Feasibility

Florence Massen<sup>1</sup>, Yves Deville<sup>1</sup>, and Pascal Van Hentenryck<sup>2</sup>

<sup>1</sup>ICTEAM, Université catholique de Louvain, Belgium,  
`florence.massen@student.uclouvain.be`

<sup>2</sup>NICTA, Australia

## Abstract

We propose an abstraction of emerging vehicle routing problems, the VRP with Black Box Feasibility. In this problem the routes of a basic VRP need to satisfy an unknown set of constraints. A black box function to test the feasibility of a route is provided. Practical examples of such problems are combinations of VRP with Loading or VRP with Scheduling. We propose a heuristic column generation-based approach to locally optimize this problem. We test the proposed approach on two applications from the literature, the Three-Dimensional Loading CVRP and the Multi-Pile VRP, showing the applicability of our approach and its good behavior compared to dedicated approaches.

**Keywords:** Column Generation, Set Partitioning, Vehicle Routing, Black Box Feasibility

Vehicle Routing Problems (VRPs) have received a great deal of attention since as early as the 1960s. While initially only basic variants have been considered, during the following decades research has focused on more complex variants, such as problems with time windows or with pick-up and delivery. Recent years have seen the emergence of rich vehicle routing problems, which strive to give a more realistic representation of problems encountered in the real world. Rich problems often require handling the combination of different complicating constraints which are typically considered individually in the literature.

In this context a new type of routing problems has emerged, problems combining routing with different combinatorial problems (e.g. combination of routing with loading (3L-CVRP [1]) or routing with scheduling (VRPTW with Driver Regulations [2])). Such problems are often tackled using very dedicated approaches. The aim of this research is to propose a generalized reformulation for this type of problem as well as an optimization procedure for the generic problem resulting from this reformulation. To do this, we

introduce the VRP with Black Box Feasibility (VRPBB). This problem is an extension of basic VRPs. Besides respecting the VRP constraints (capacity, time windows, ...) each route needs to verify an unknown set of constraints  $F$ . The feasibility of a route with respect to  $F$  is verified using a deterministic black box algorithm. This algorithm is considered of non-linear time-complexity in the length of the route.

The VRPBB can thus be instantiated to a specific problem by simply plugging an appropriate feasibility check function. An approach optimizing the VRPBB is thus generic with respect to the black box function, and therefore generic with respect to the combinatorial problem considered besides routing.

We propose to reformulate the VRPBB as a Set Partitioning Problem which we address using a (non-exact) column generation-based approach. At each iteration a set of columns is heuristically generated and collected by Collector Ants. Next the relaxation of a Set Covering Problem is solved on the column pool. The Collector Ants are guided by pheromone deposits on the edges of the problem graph. Those pheromones are updated each iteration in function of the current relaxed solution. The approach allows us to iteratively improve the lower bound of the considered problem. An integer solution is found by solving the Set Partitioning Problem on the set of collected columns using a MIP solver.

The proposed approach is tested on two different problems, the Three-dimensional Loading CVRP [1] and the Multi-Pile VRP [3]. In both cases the algorithms used as black box feasibility check stem from the current state-of-the-art.

The contribution of this research is three-fold. First we propose a new generic problem, the Vehicle Routing Problem with Black Box Feasibility, which allows to represent Vehicle Routing Problems demanding the feasibility of a combinatorial problem per route. Second, we propose an algorithm to solve this generic problem. Our method works thus independently of the combinatorial problem to be solved. It can easily be applied to a new problem by plugging an appropriate black box function. Finally we demonstrate the applicability of the proposed method on two problems, and compare our results with those of existing dedicated approaches, to show that our approach is highly competitive.

## References

- [1] M. Gendreau, M. Iori, G. Laporte and S. Martello, A Tabu Search Algorithm for a Routing and Container Loading Problem, *Transportation Science* Vol. 40, 2006
- [2] E. Prescott-Gagnon, G. Desaulniers, M. Drexler and L.-M. Rousseau, European Driver Rules in Vehicle Routing with Time Windows, *Transportation Science* Vol. 44, 2010
- [3] K. Doerner, G. Fuellerer, M. Gronalt, R. Hartl and M. Iori, Metaheuristics for the Vehicle Routing Problem with Loading Constraints, *Networks* Vol. 49, 2007