

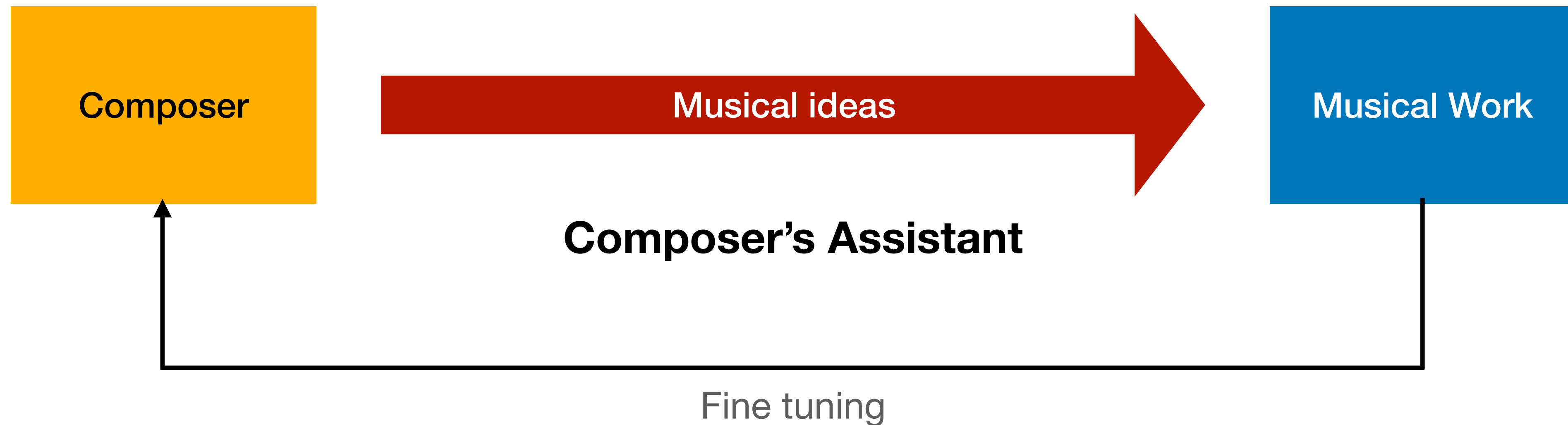


A constraint formalization of Fux's counterpoint

Damien Sprockeels, Thibault Wafflard, Peter Van Roy and Karim Haddad, JIM 2023

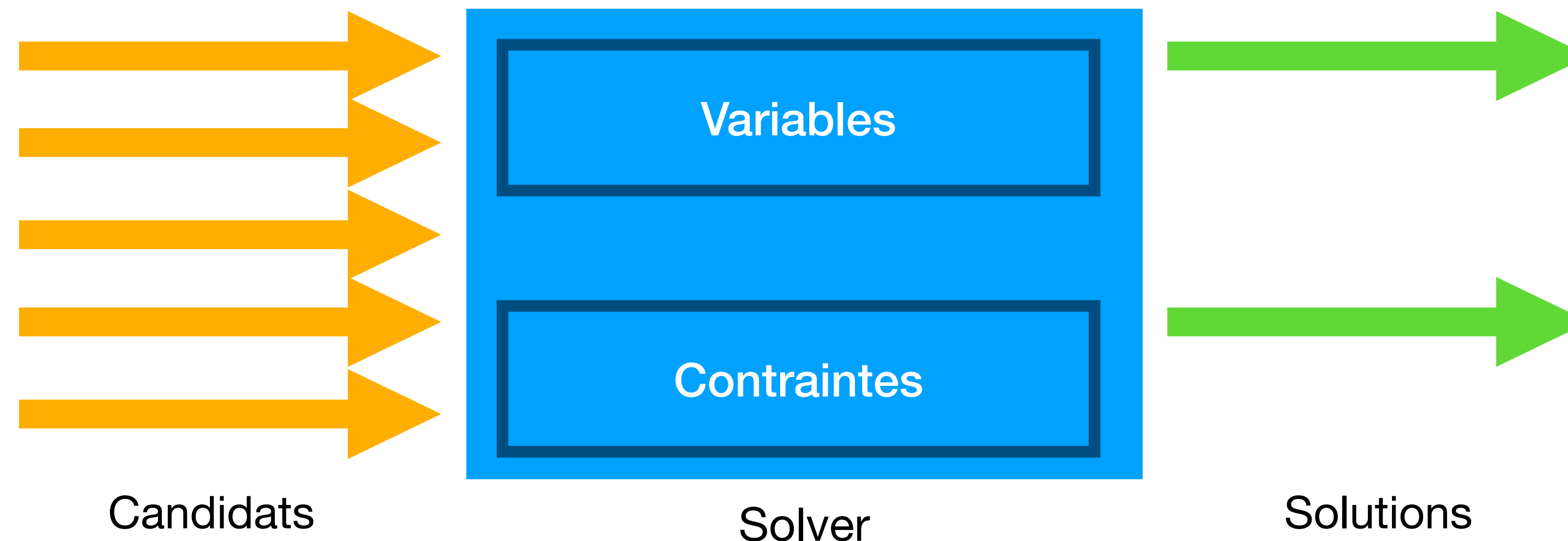
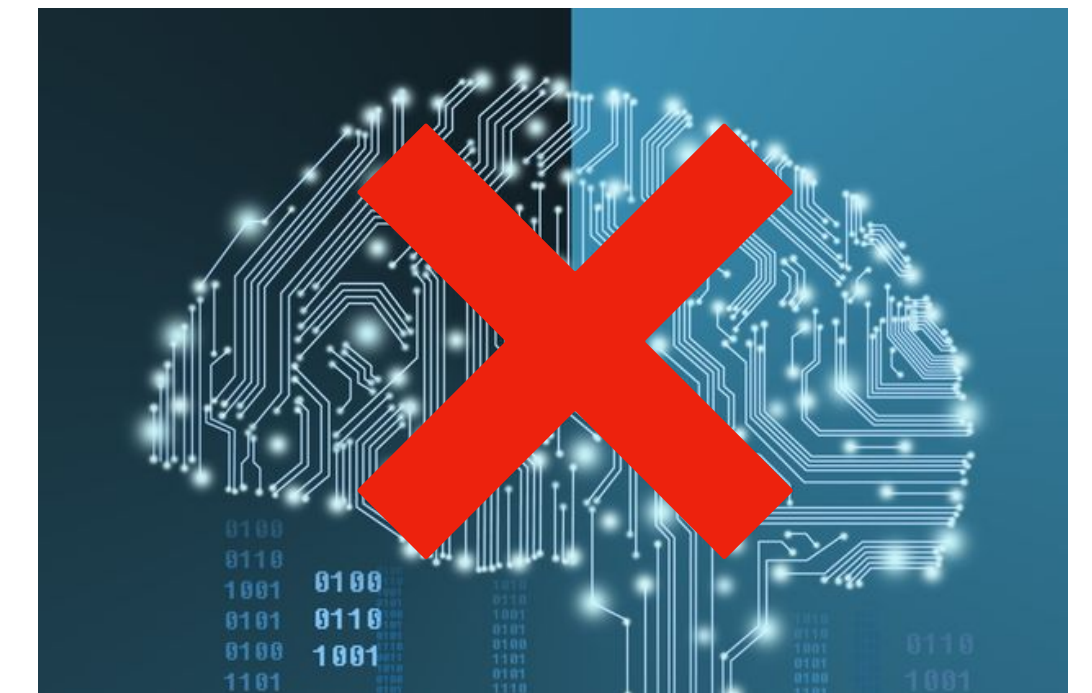
Context

- End goal : Composer's assistant
 - Modeling a (many) complete style(s)
 - No programming skills required
 - Automate repetitive tasks



Context

- End goal : Programming free composer assistant
- Constraint programming
 - Not machine learning because we want transparency
 - Powerful paradigm when used carefully



Context

- End goal : Programming free composer assistant
- Why constraint programming?
- This work : Counterpoint writing tool
 - Polyphonic musical style
 - Melodically independent voices
 - Harmonically interdependent
 - Gradus ad Parnassum, J.J. Fux



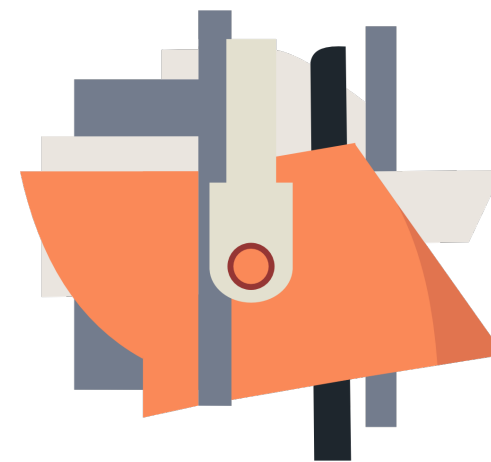
The image displays a musical score and a counterpoint tool interface. The top part shows a musical score with two staves. The first staff is in 4/4 time, marked with a tempo of 120. The second staff is in 4/4 time, marked with a tempo of 120. The score consists of ten measures, each containing a single note. The notes are: 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11. The bottom part shows a counterpoint tool interface. It features a grid with a vertical axis labeled C5, C4, C3, and C2, and a horizontal axis labeled 1 through 10. The grid contains various colored rectangles representing notes and rests. The interface also includes a sidebar with controls for Signature (4 / 4), Groove, None, Pgm Change, Bank, Sub, and Pgm 1.

Context

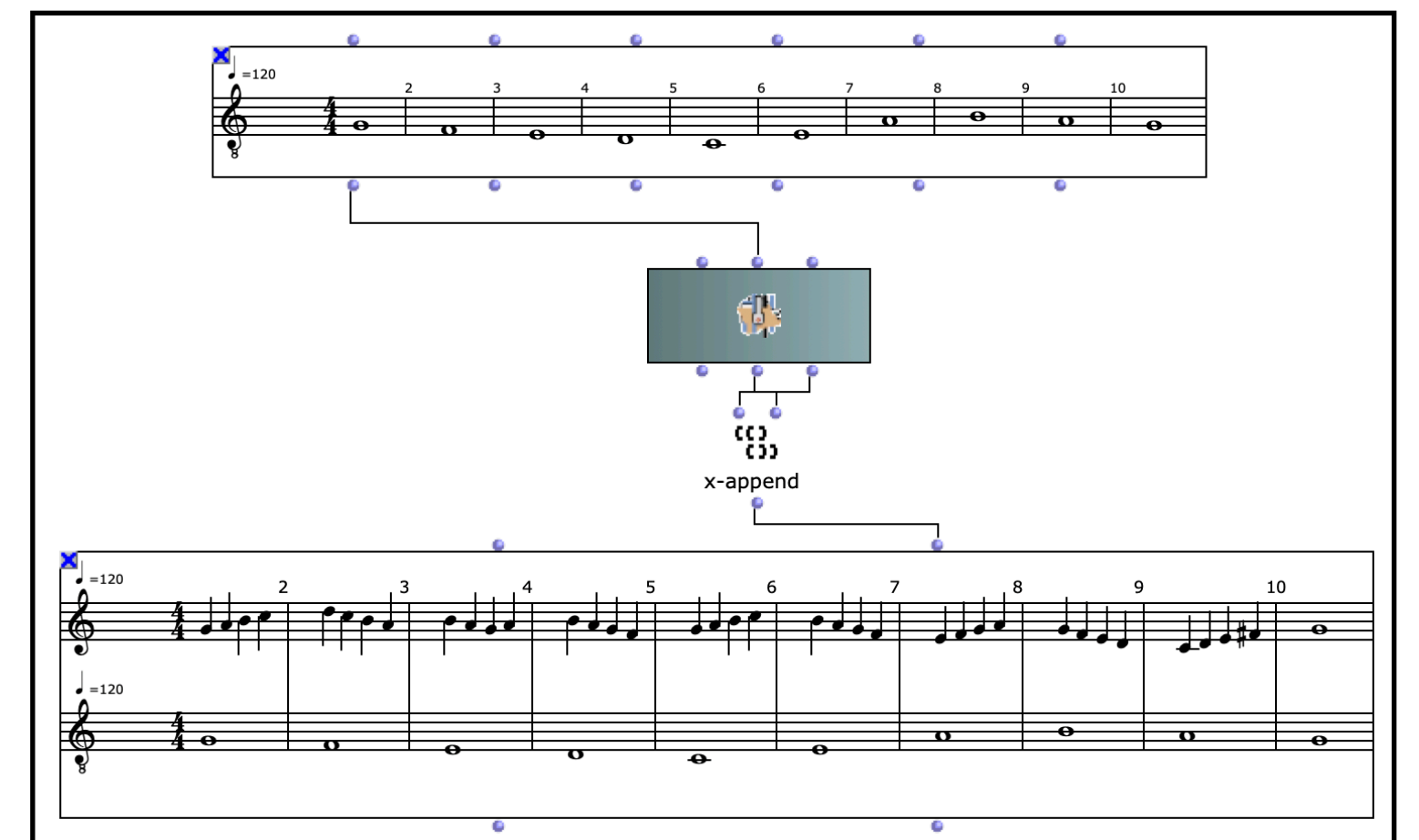
- This work : Counterpoint writing tool
- End goal : Programming free composer assistant
- What is counterpoint?
- Why constraint programming?



Gecode



OM

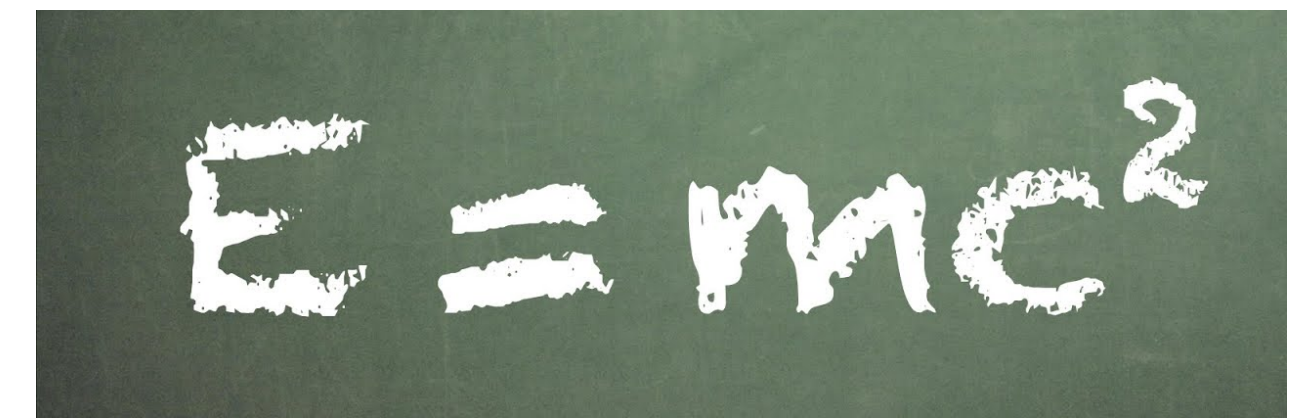


Mathematical formalization

Constants and variables

- How to go from music theory to math?



The equation $E = mc^2$ is written in white chalk on a dark green chalkboard, representing mathematical formalization.

Mathematical formalization

Constants and variables

- Input
 - Cantus firmus: Cf
 - Number of notes: m



Mathematical formalization

Constants and variables

- Input
 - Cantus firmus: Cf
 - Number of notes: m
- Output
 - Counterpoint: Cp
 - Number of notes: n



Mathematical formalization

Constants and variables

- Intervalles mélodiques: M



Mathematical formalization

Constants and variables

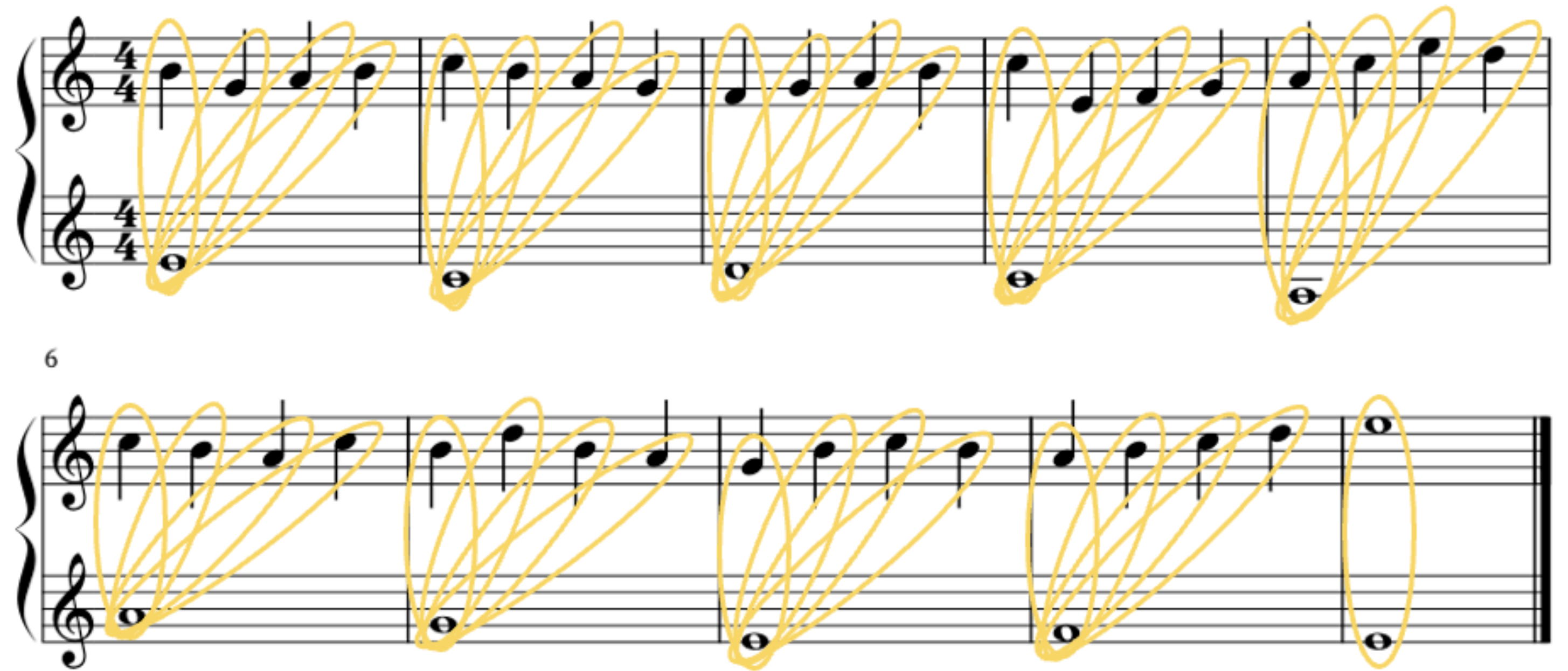
- Intervalles mélodiques: M
- Mouvements: P

The image displays two systems of musical notation in 4/4 time. Each system consists of a grand staff with a treble and bass clef. Red arrows are drawn across the staves to highlight specific melodic intervals and movements. In the first system, the top staff contains a sequence of eighth and quarter notes, while the bottom staff contains whole notes. Red arrows connect notes between the two staves, indicating intervals. In the second system, the top staff continues the melodic line, and the bottom staff contains whole notes. A red arrow connects the first note of the top staff to the first note of the bottom staff, indicating a movement. The number '6' is written below the first system.

Mathematical formalization

Constants and variables

- Intervalles mélodiques: M
- Mouvements: P
- Intervalles harmoniques : H



Mathematical formalization

All harmonic intervals on the first beat must be consonances

- $H[j,1] \in Cons$

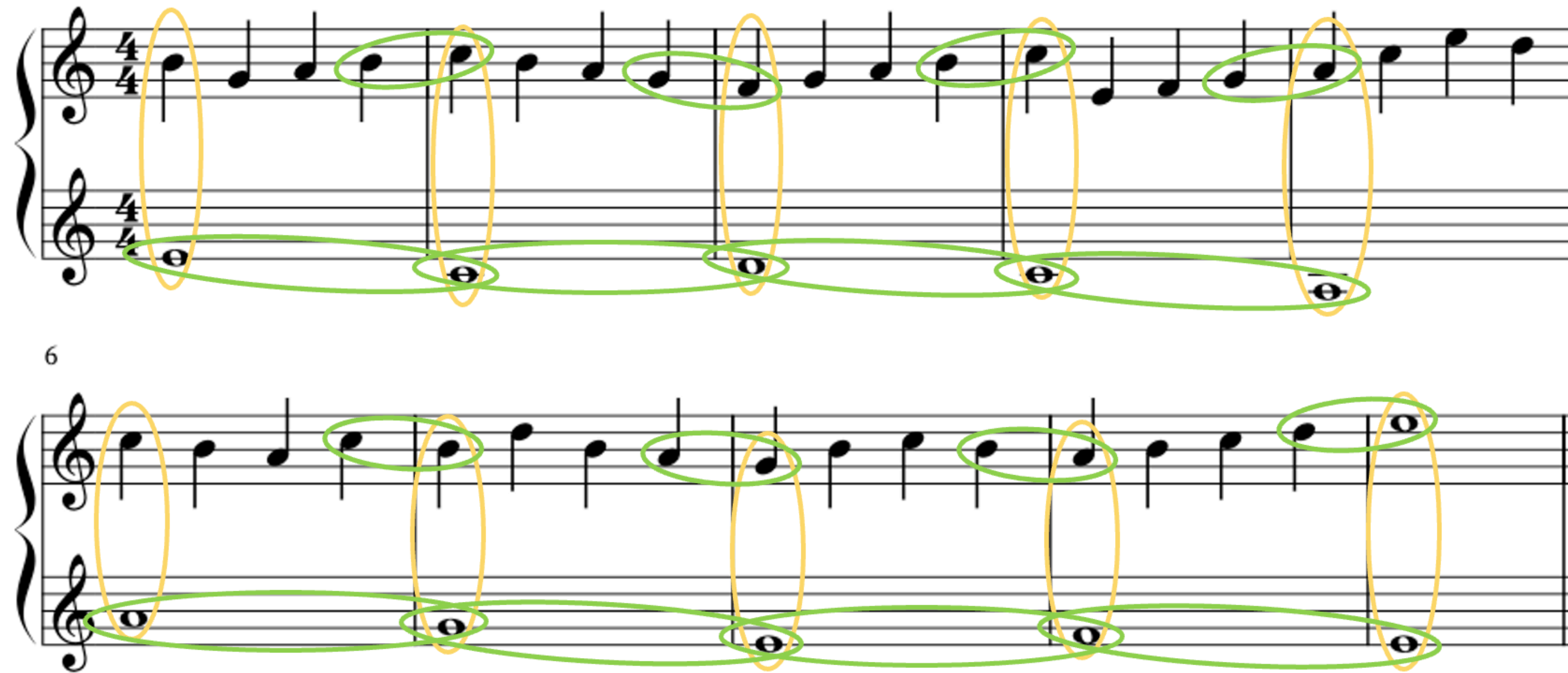


Notes impacted by this rule

Mathematical formalization

Perfect consonances can't be reached by direct motion

- $\forall l \in [1, m - 1] \ H[l + 1, 1] \in \text{Cons}_p \implies P[l] \neq 2$

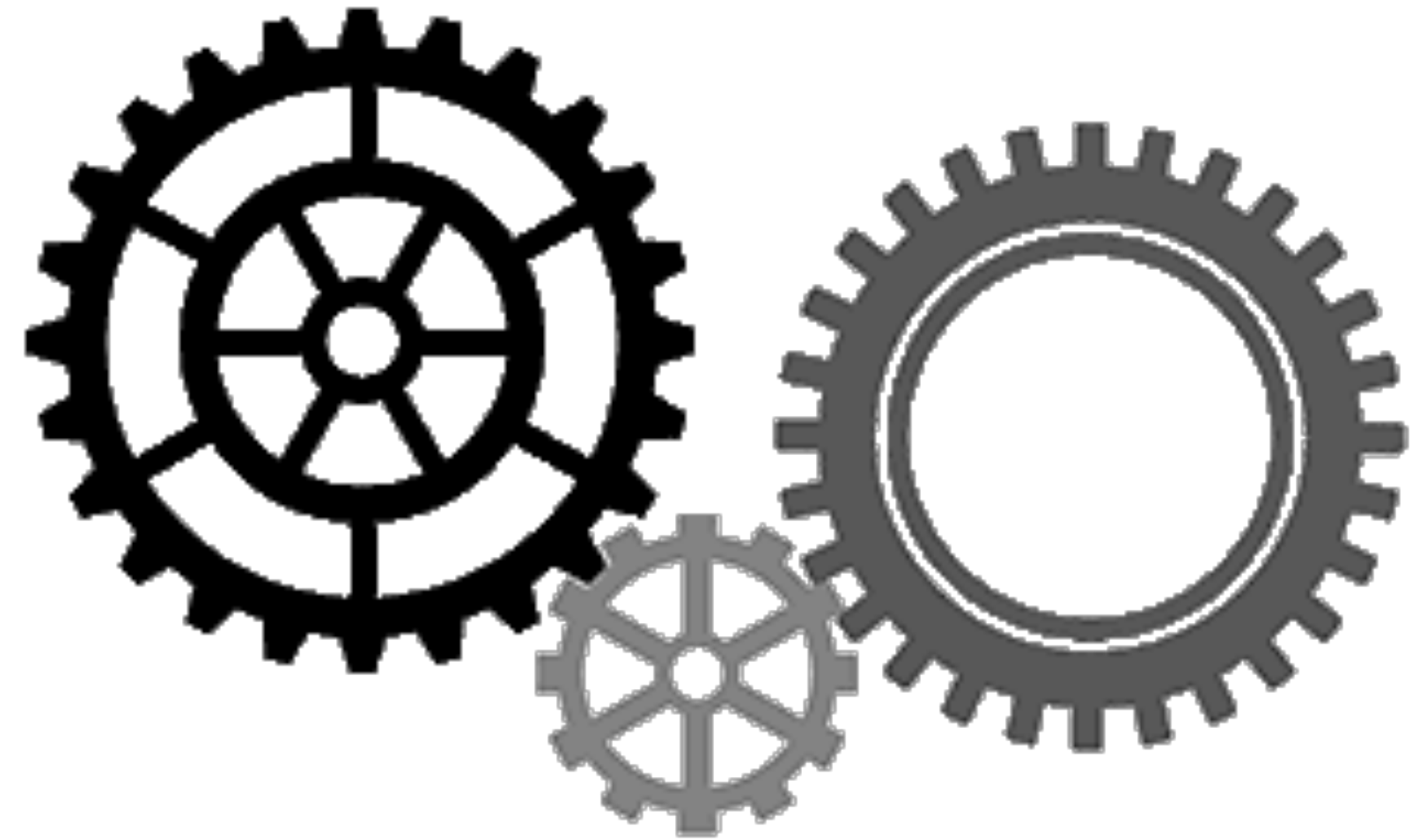


Notes impacted by this rule

Implementation

How the solver works

- From mathematical rules to constraints
- Preference-based system (customizable)
- Cost minimization to respect preferences
- Different from human approach (based on degree and domain size)



First species (generated by the solver)

16

Implementation and examples

Third species (generated by the solver)

Counterpoint in F generated by our solver

(Parolier) (Sous-titre) (Compositeur)

$\text{♩} = 120$

7

Implementation and examples

Fifth species (after modification, generated by the solver)

The image displays a screenshot of a music notation software interface, likely MuseScore, showing a fifth species counterpoint example. The top section features a piano roll with multiple staves, each containing a sequence of notes (e.g., 66666666, 77777777, 88888888, 99999999, 1000000000, 1111111111, 222222222, 3333333333, 4444444444, 5555555555). The main workspace shows a piano roll with a grid of notes and rests, organized into measures. The notes are labeled with letters (A, G, F, E, D, C) and numbers (3, 2, 1, 0) indicating their position in the counterpoint. The interface includes a sidebar with controls for Signature (4/4), Groove, and Pgm Change. The bottom status bar shows the current measure (149) and tempo (110.00).

Implementation and examples

Fifth species (chromatic bass line, generated by the solver)

2 3 4 5 6 7 8 9

The image displays a musical score and a corresponding piano roll for a Fifth species counterpoint exercise. The score is written in 4/4 time and consists of two staves. The upper staff features a treble clef and a key signature of one sharp (F#), with a melodic line that begins with a whole rest and is followed by eighth and quarter notes. The lower staff features a bass clef and the same key signature, with a chromatic bass line consisting of whole notes. Above the staves, measure numbers 2 through 9 are indicated. Below the staves, a piano roll visualization shows the pitch and duration of the notes for both staves across the measures. The piano roll interface includes a vertical piano keyboard on the left with labels C2, C3, C4, and C5. A control panel on the far left contains buttons for 'None', 'Pgm Change', 'Commit', 'Bank ---', 'Sub ---', and 'Pgm 1'. The piano roll itself has a dark gray background with a grid, and notes are represented by horizontal bars with labels indicating their pitch (e.g., D4, C#4, A3, G3, F3, E3, D3, A2, G#2, G2, F#2, F2, E2, C3, B2, A2).

Conclusion

- Work in progress
- Very comprehensive formalization required for constraint programming to be relevant
- Additional constraints on melodic development (long range) for the counterpoint are necessary to have more interesting melodies
- It can be extended to more complex styles than counterpoint

