UCLouvain

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A constraint formalization of Fux's counterpoint

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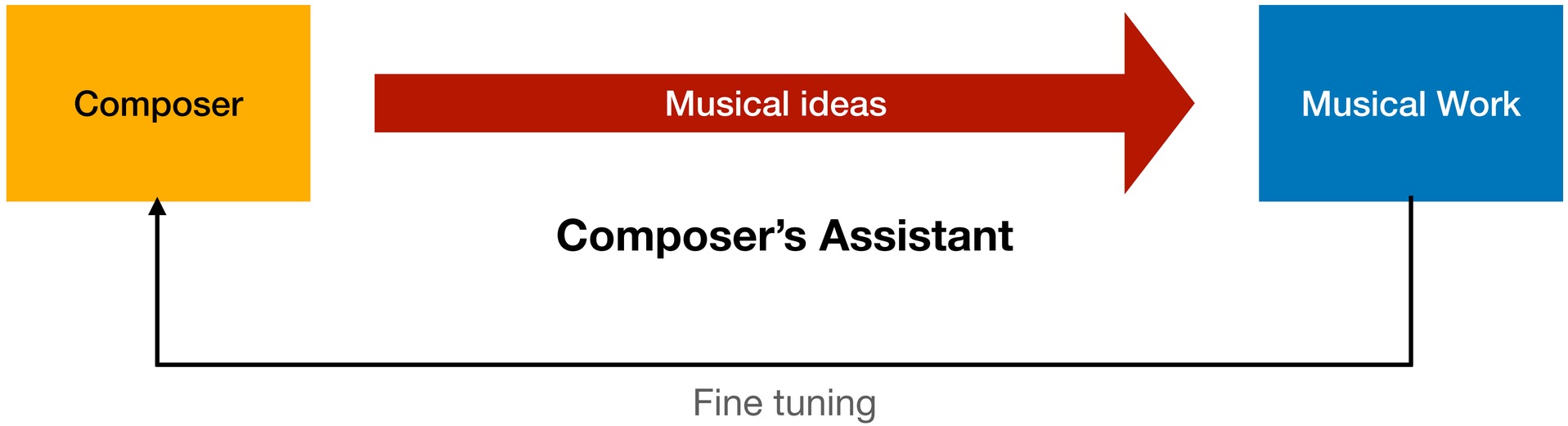
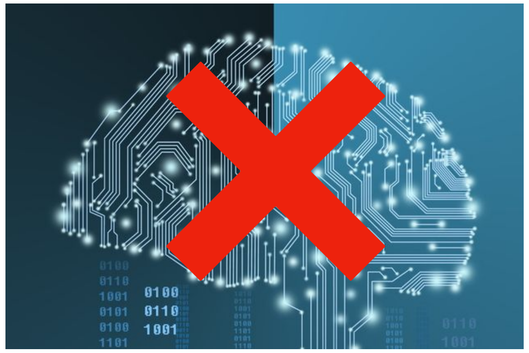
Context

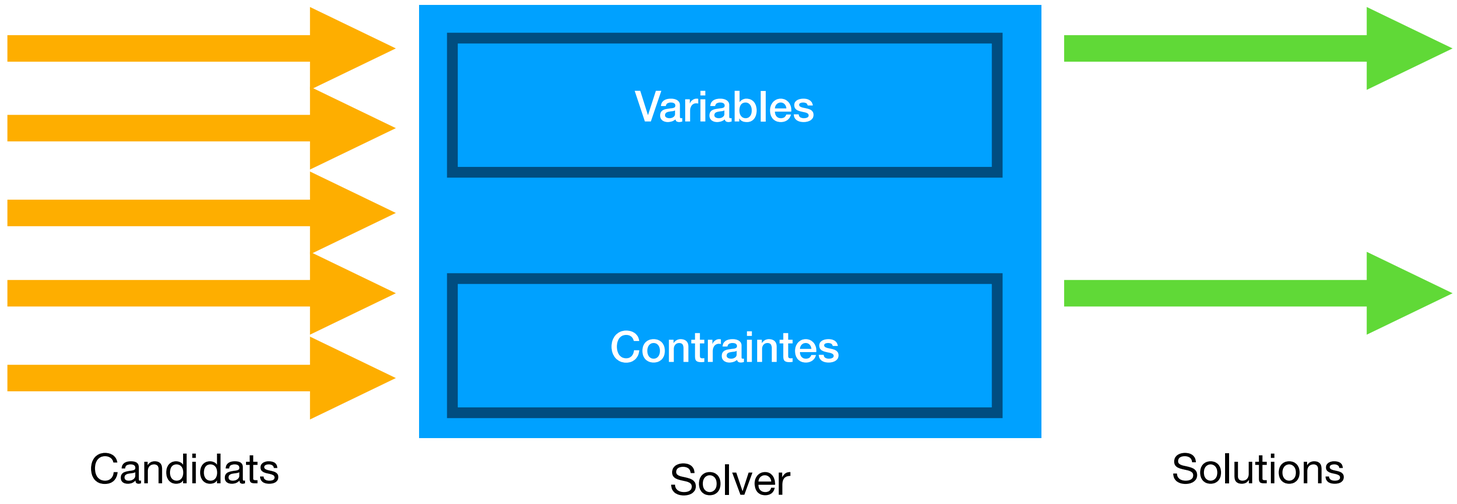
- End goal : Composer’s assistant
  - Modeling a (many) complete style(s)
  - No programming skills required
  - Automate repetitive tasks



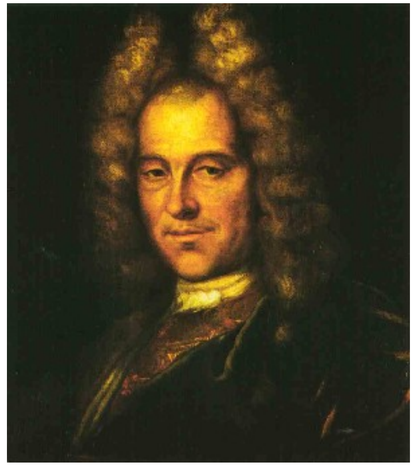
Context

- End goal : Programming free composer assistant
- Constraint programming
  - Not machine learning because we want transparency
  - Powerful paradigm when used carefully

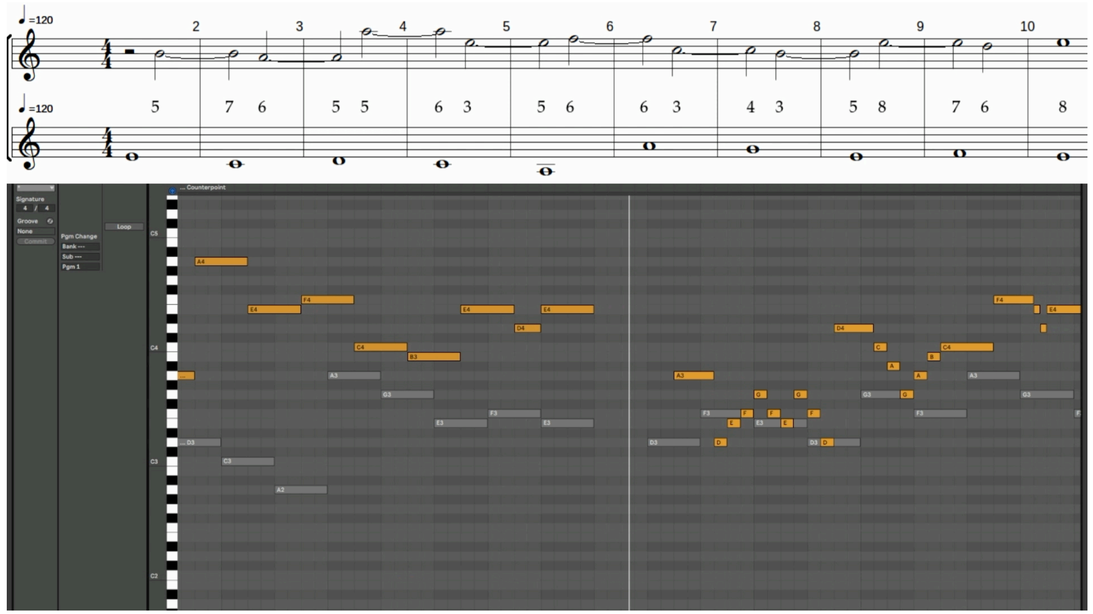




Context

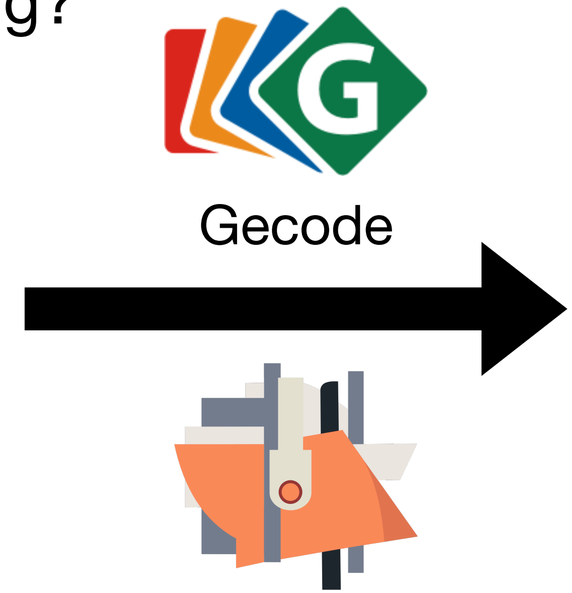


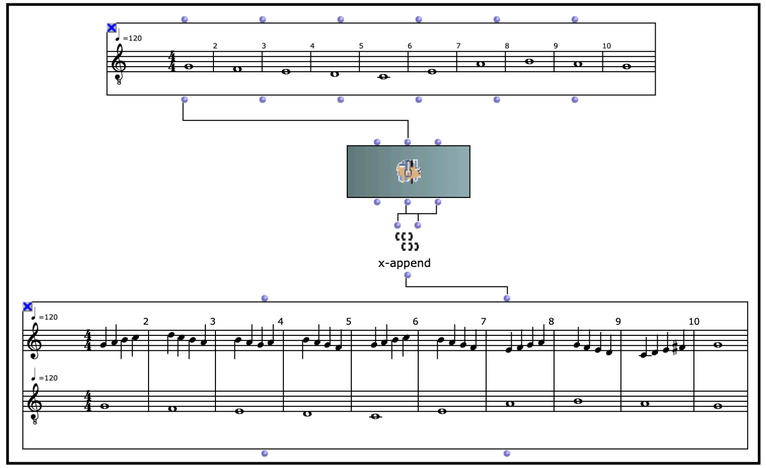
- End goal : Programming free composer assistant
- Why constraint programming?
- This work : Counterpoint writing tool
  - Polyphonic musical style
    - Melodically independent voices
    - Harmonically interdependent
  - Gradus ad Parnassum, J.J. Fux



Context

- This work : Counterpoint writing tool
- End goal : Programming free composer assistant
- What is counterpoint?
- Why constraint programming?

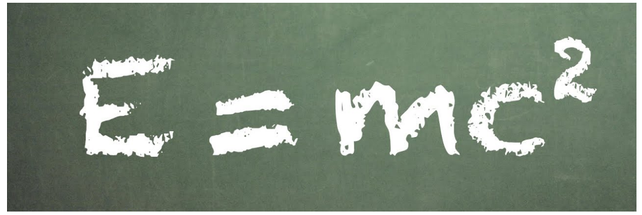




Mathematical formalization

Constants and variables

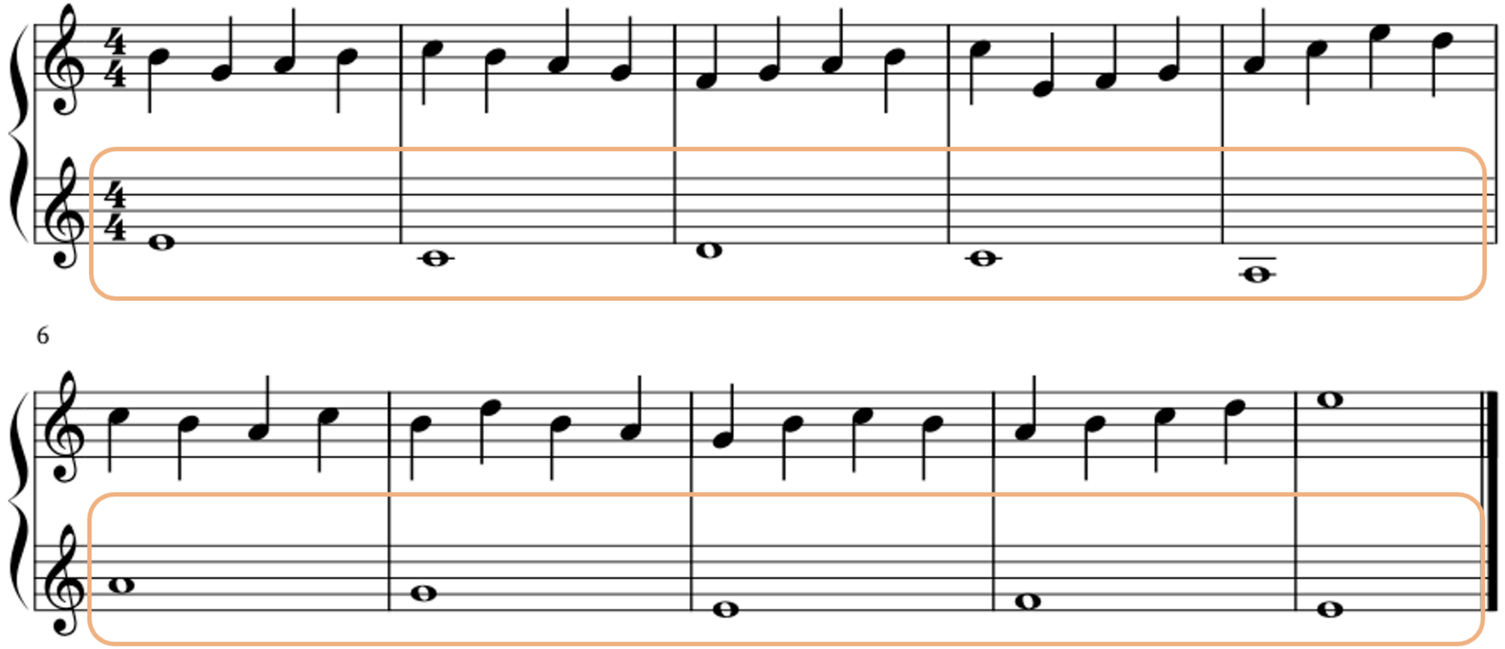
- How to go from music theory to math?



Mathematical formalization

Constants and variables

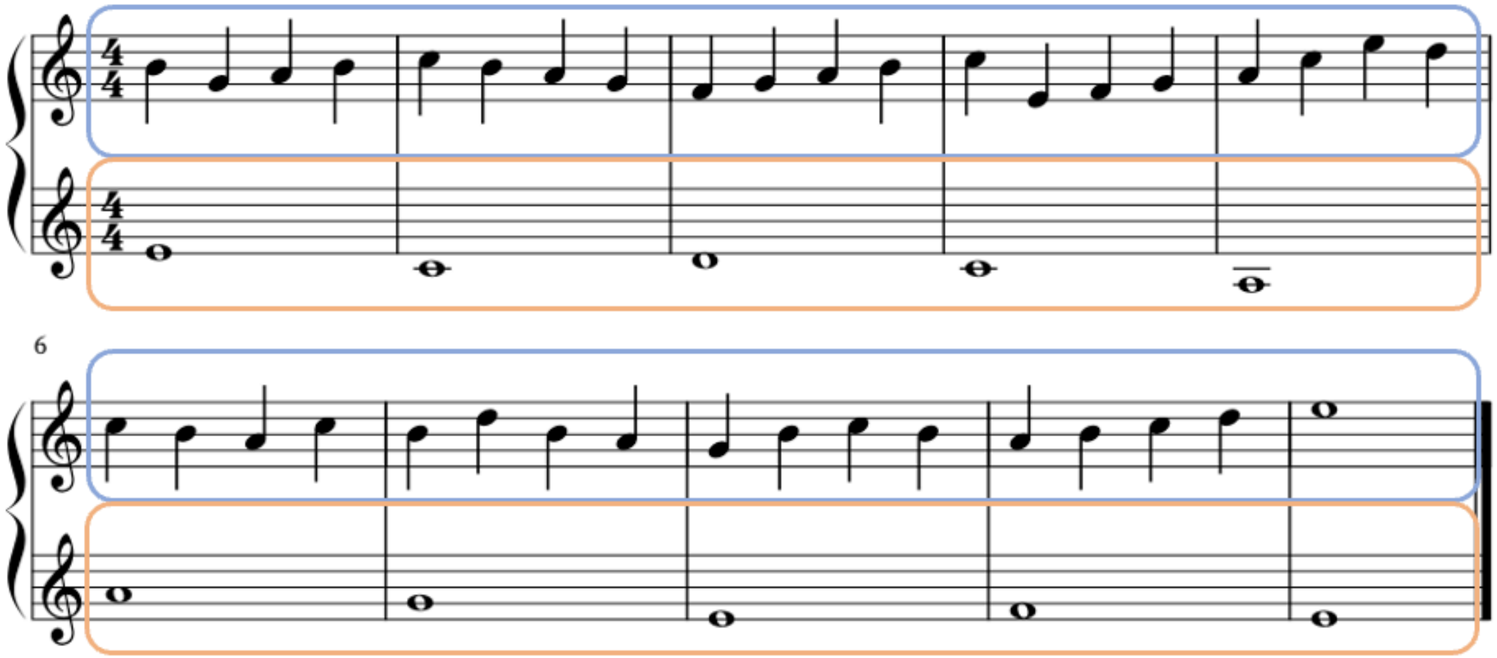
- Input
  - Cantus firmus: Cf
  - Number of notes: m



Mathematical formalization

Constants and variables

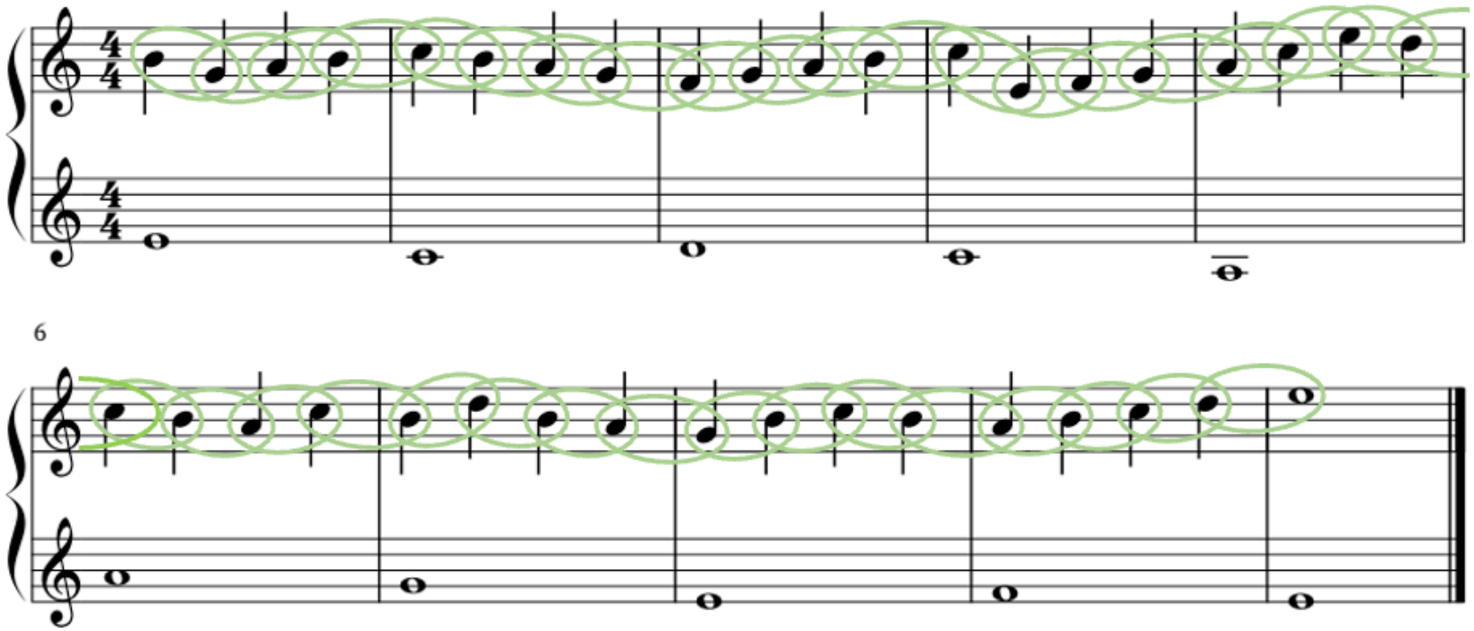
- Input
  - Cantus firmus: Cf
  - Number of notes: m
- Output
  - Counterpoint: Cp
  - Number of notes: n



Mathematical formalization

Constants and variables

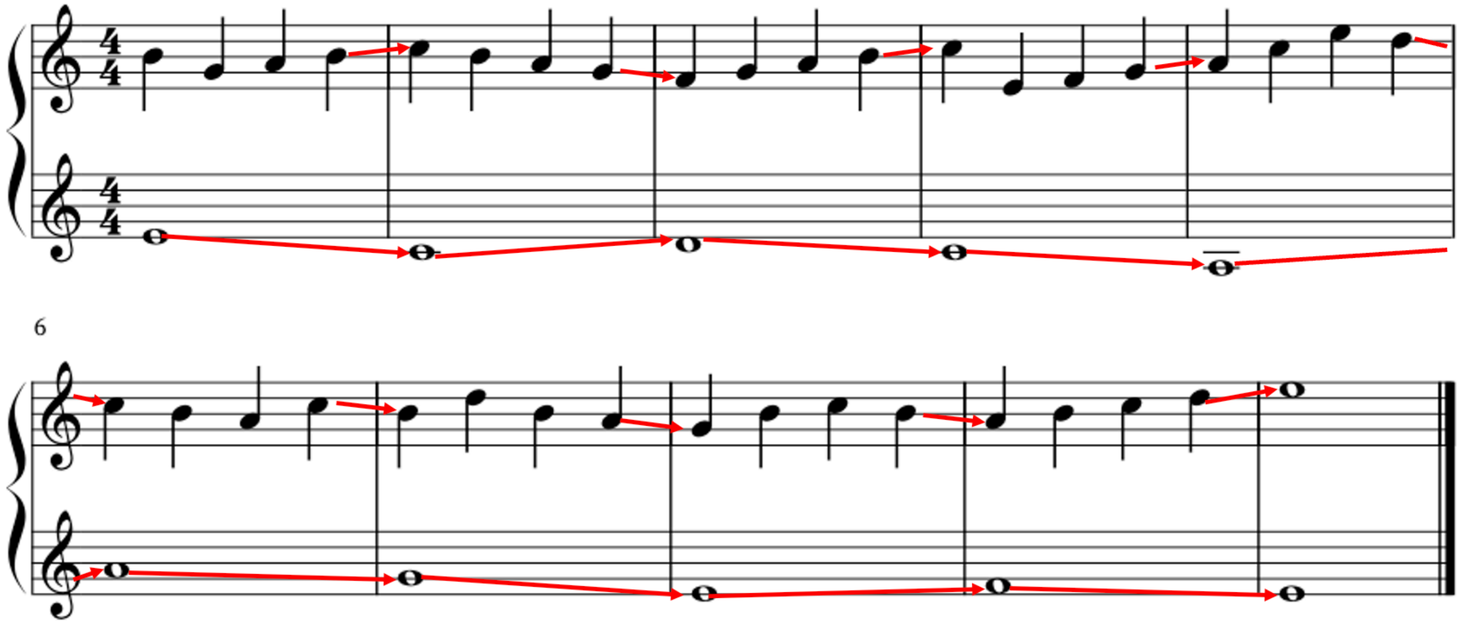
- Intervalles mélodiques: M



Mathematical formalization

Constants and variables

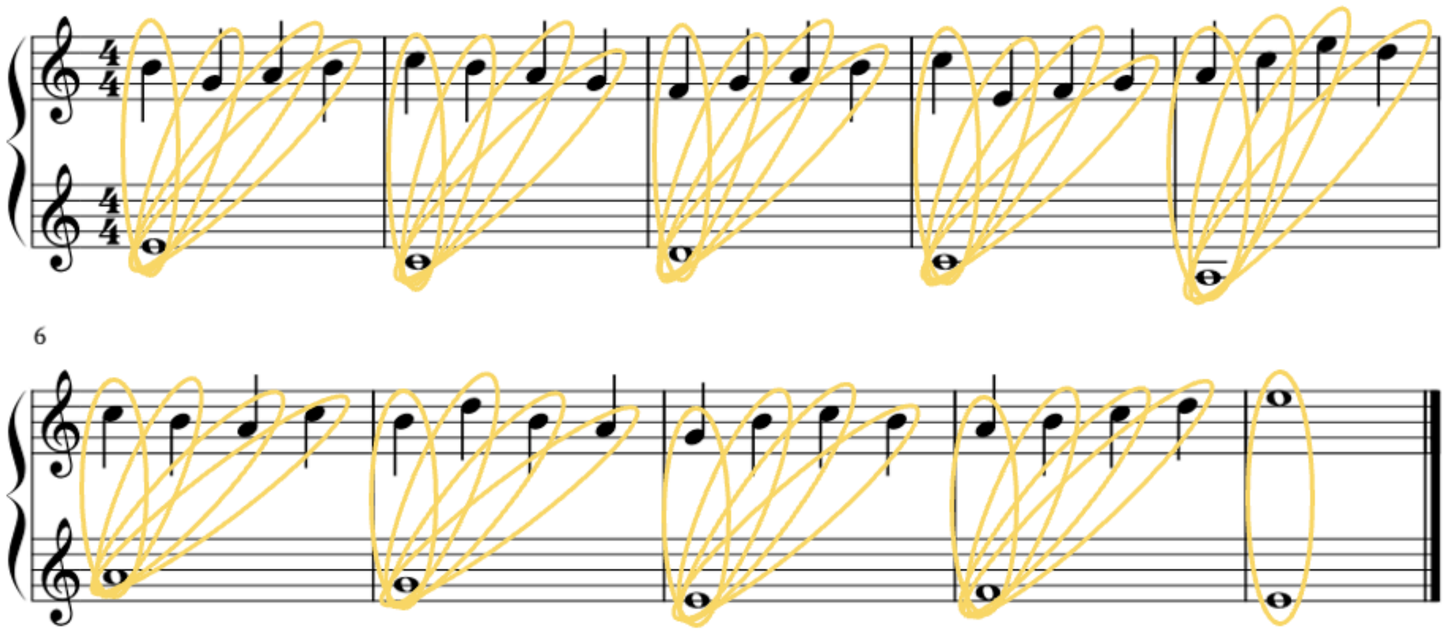
- Intervalles mélodiques: M
- Mouvements: P



Mathematical formalization

Constants and variables

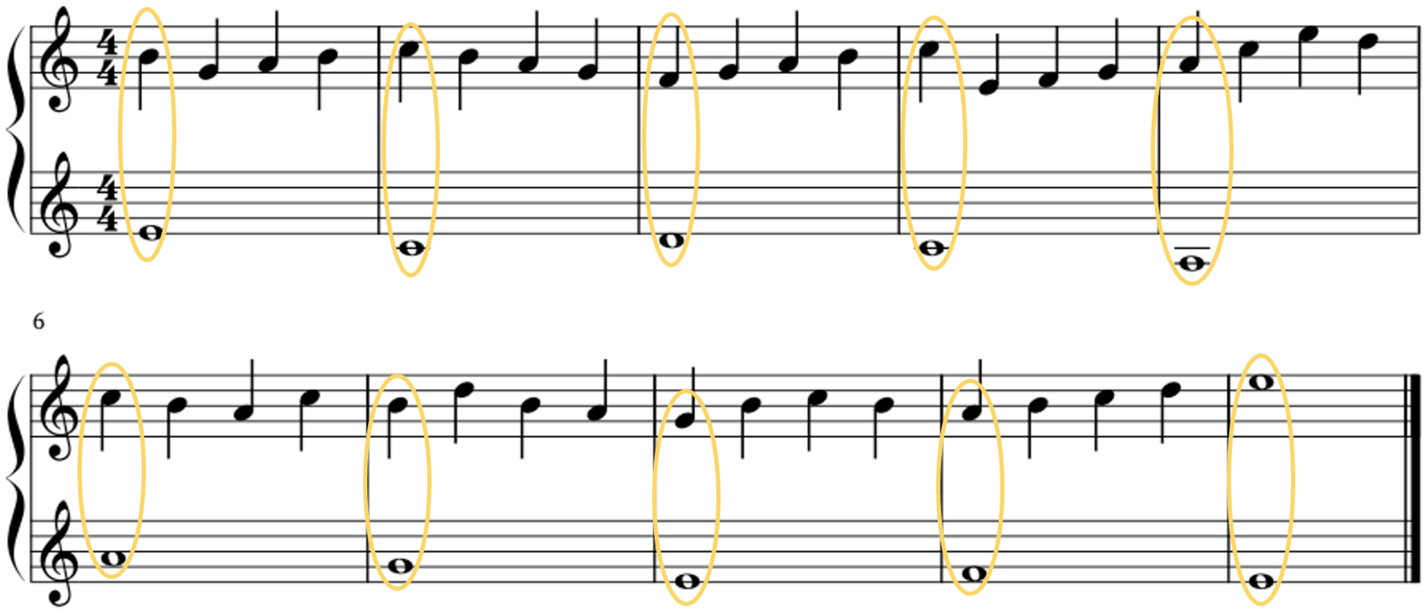
- Intervalles mélodiques: M
- Mouvements: P
- Intervalles harmoniques : H



Mathematical formalization

All harmonic intervals on the first beat must be consonances

- $H[j,1] \in Cons$



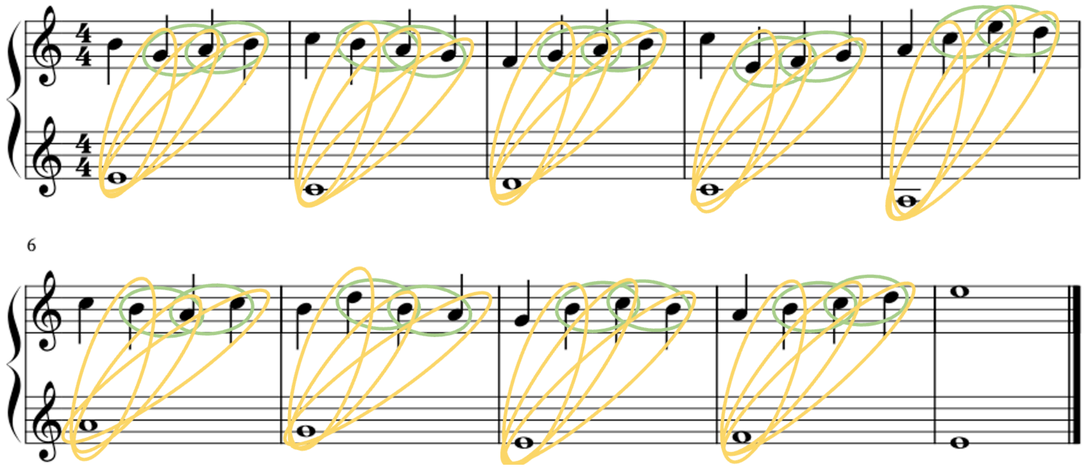
Notes impacted by this rule

Mathematical formalization

Third note of a measure is dissonant implies diminution

- $H[j,3] \notin Cons \implies$

$$H[j,2] \in Cons \wedge H[j,4] \in Cons$$
$$|Cp[4(j-1)+4] - Cp[4(j-1)+3]| \leq 2$$
$$|Cp[4(j-1)+3] - Cp[4(j-1)+2]| \leq 2$$
$$Cp[4(j-1)+4] > Cp[4(j-1)+3] > Cp[4(j-1)+2] \vee$$
$$Cp[4(j-1)+4] < Cp[4(j-1)+3] < Cp[4(j-1)+2]$$

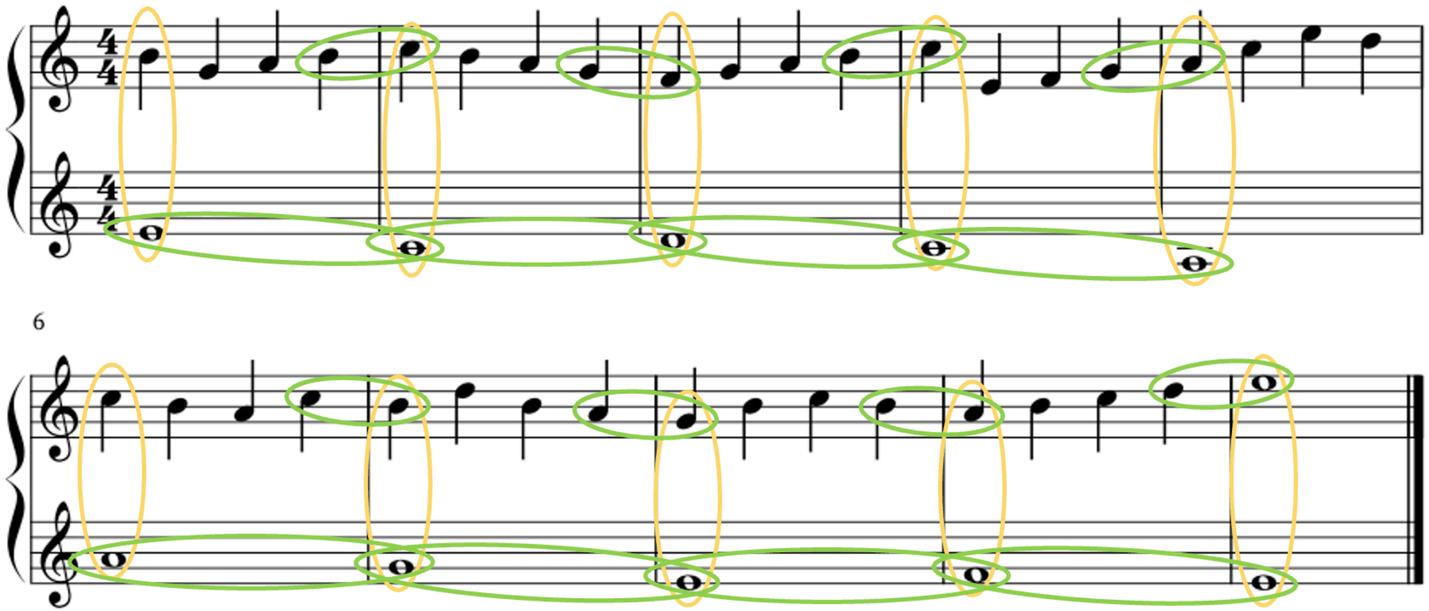


Notes impacted by this rule

Mathematical formalization

Perfect consonances can’t be reached by direct motion

- $\forall l \in [1, m-1]\ H[l+1,1] \in Cons_p \implies P[l] \neq 2$

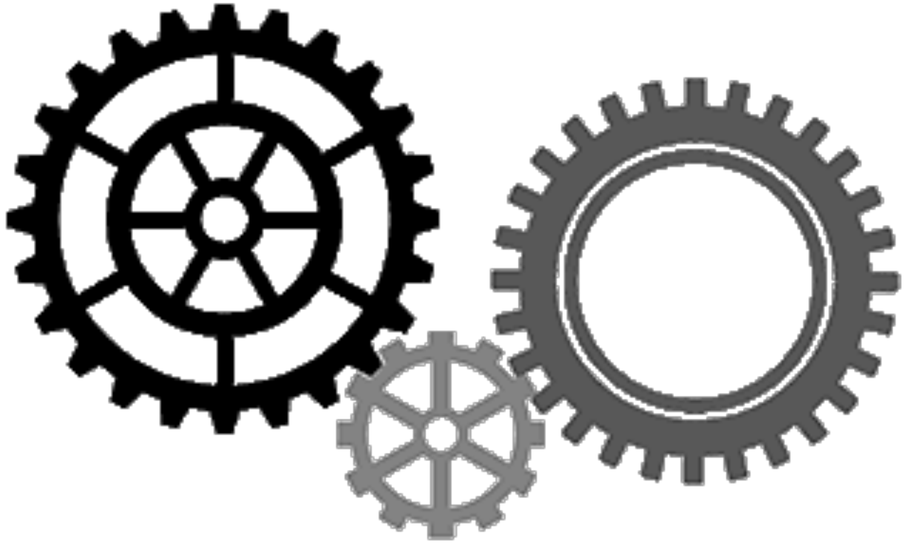


Notes impacted by this rule

Implementation

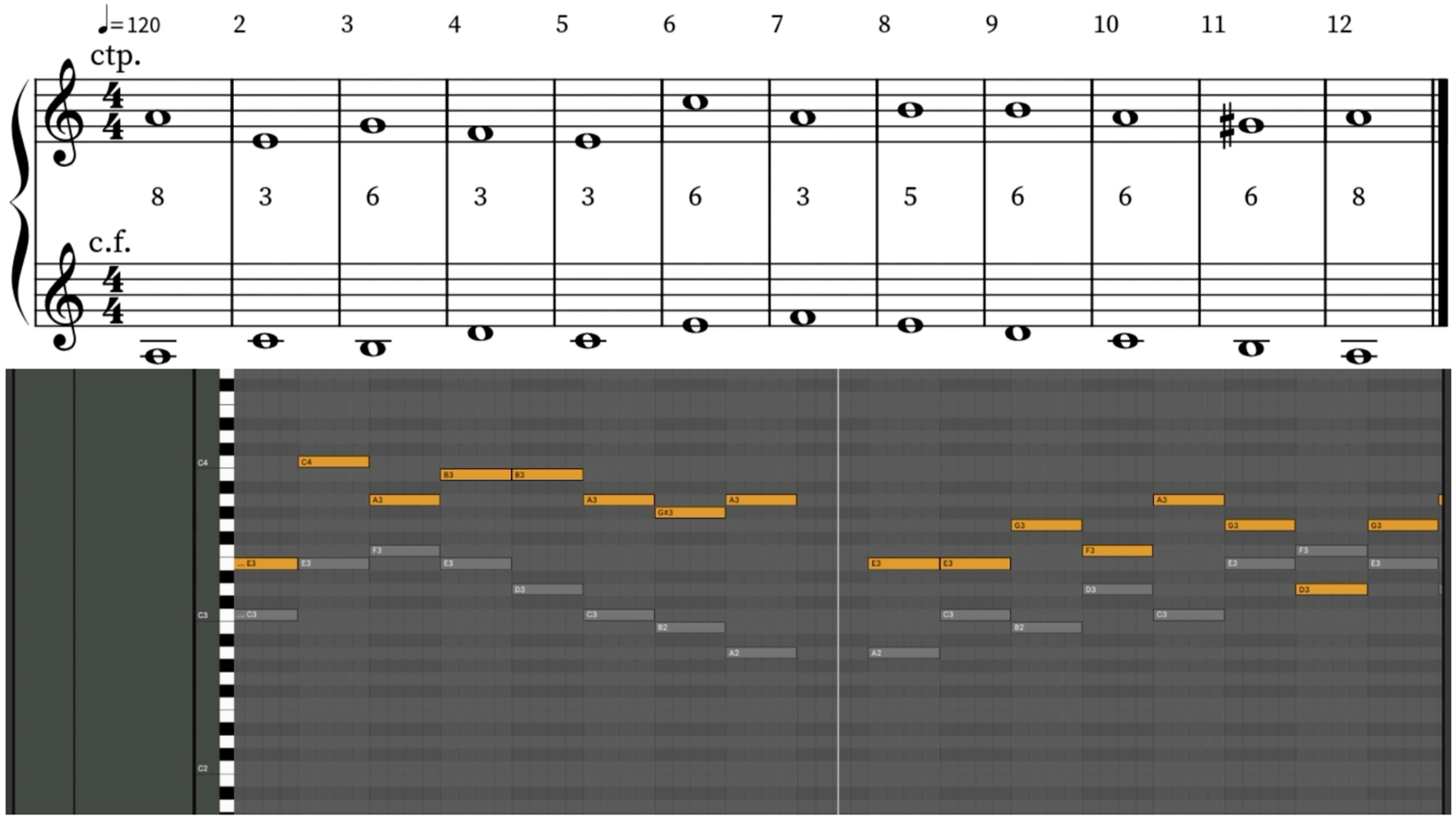
How the solver works

- From mathematical rules to constraints
- Preference-based system (customizable)
- Cost minimization to respect preferences
- Different from human approach
  (based on degree and domain size)



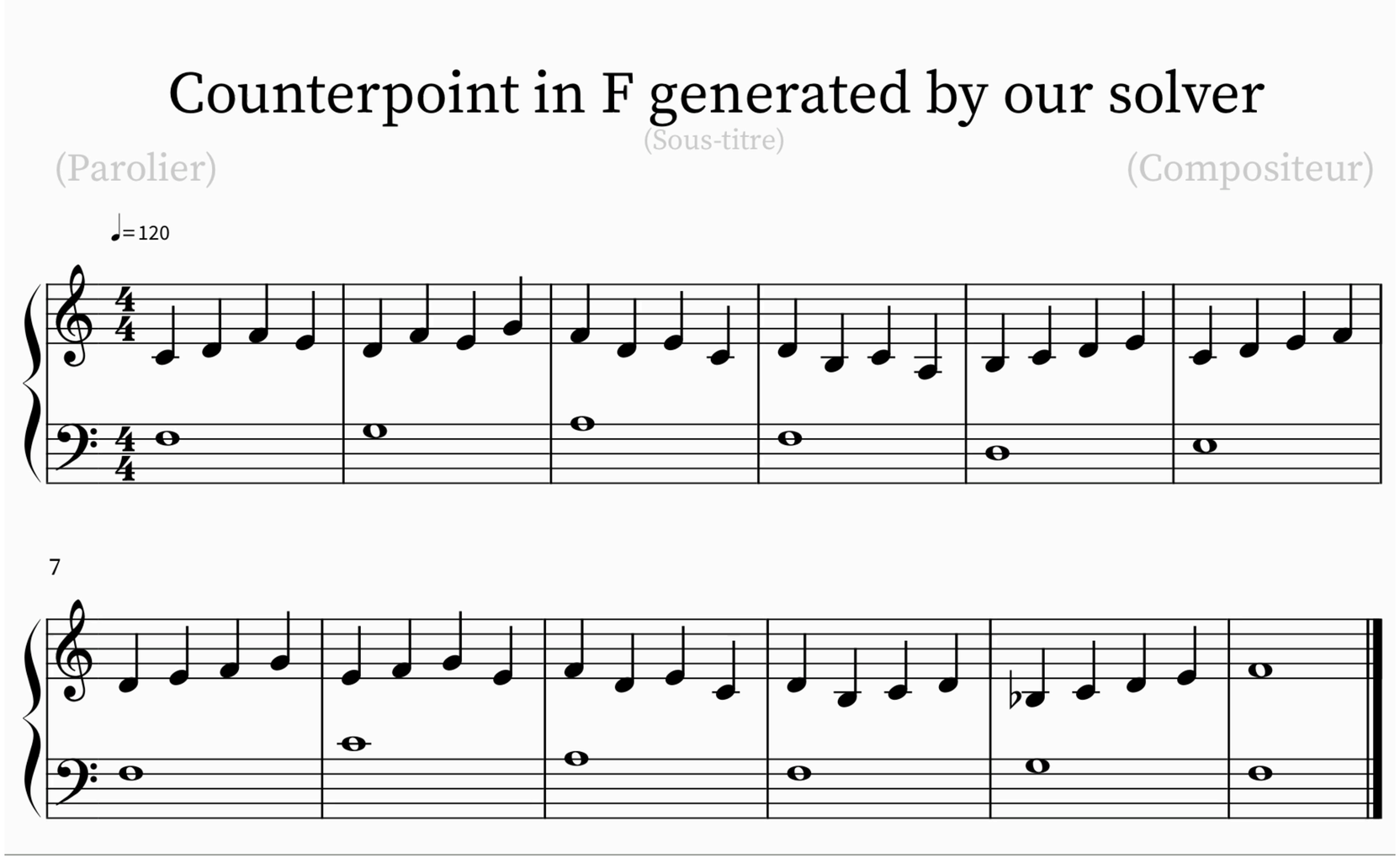
Implementation and examples

First species (generated by the solver)



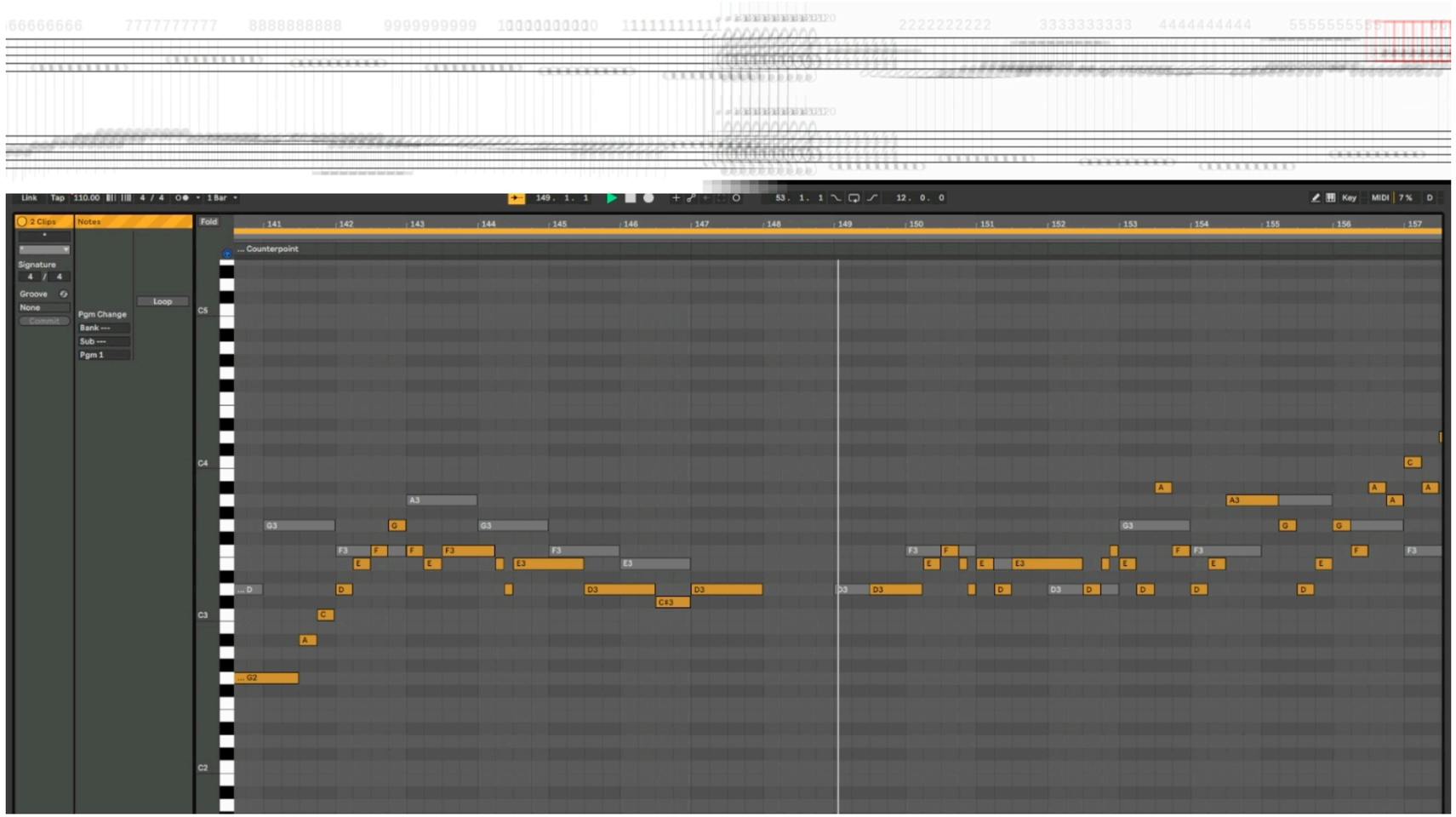
Implementation and examples

Third species (generated by the solver)



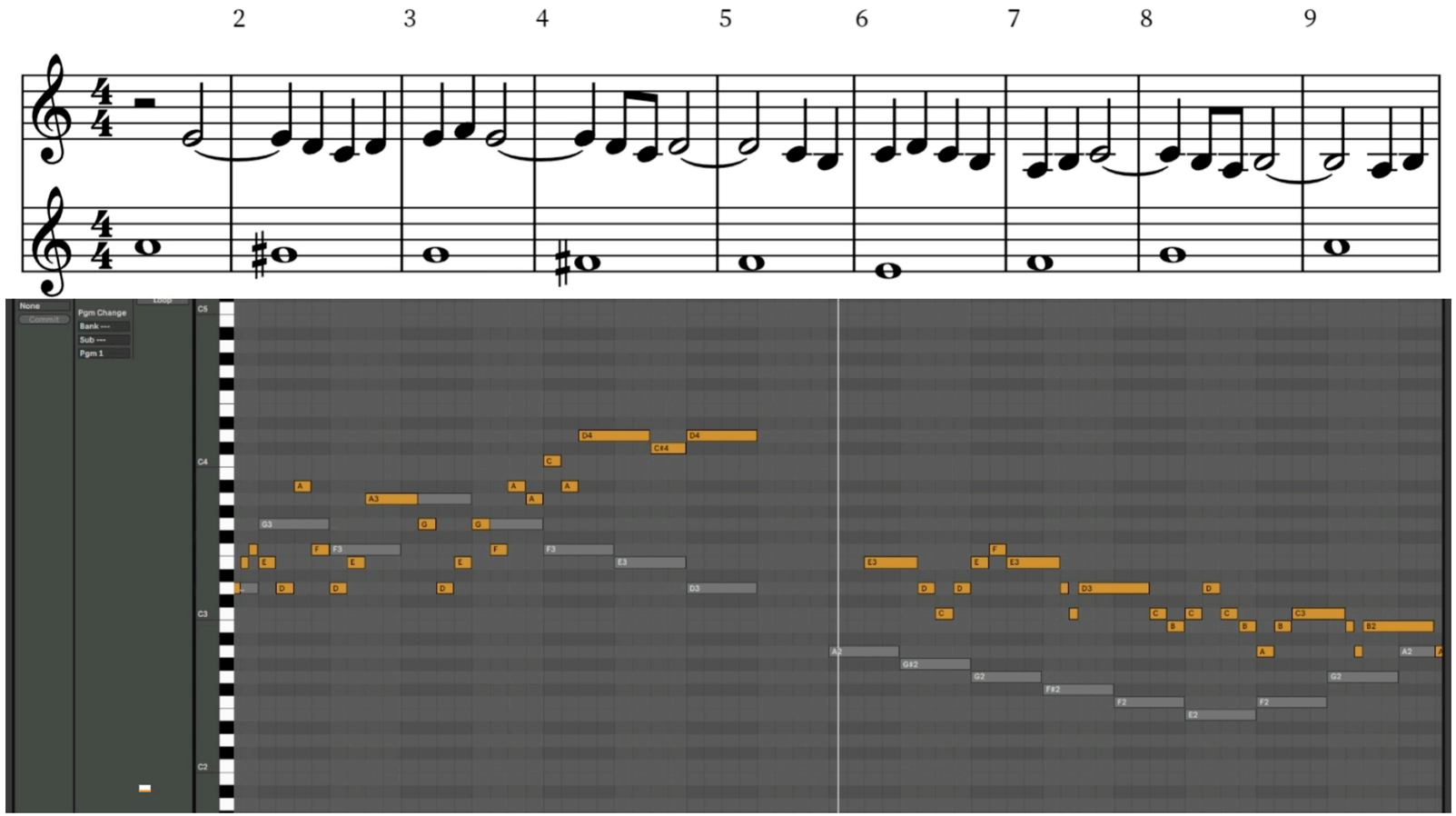
Implementation and examples

Fifth species (after modification, generated by the solver)



Implementation and examples

Fifth species (chromatic bass line, generated by the solver)



Conclusion

- Work in progress
- Very comprehensive formalization required for constraint programming to be relevant
- Additional constraints on melodic development (long range) for the counterpoint are necessary to have more interesting melodies
- It can be extended to more complex styles than counterpoint